THE REPRESENTATION OF TINTS AND SHADES OF COLORS BY MEANS OF ROTATING DISCS.

By A. Kirschmann.

In a former article of this Journal I gave, on the basis of a mathematical deduction, directions for the construction of discs which show, when in rotation, all saturation degrees of a color-tone with exclusion of differences in intensity (Am. Journ. of Psy., Vol. VII, p. 386 ff.).[1] It may sometimes

[1] I am sorry to state that by some mistake of the printers, the formulæ which I gave in that paper were distorted almost beyond recognition. I obtained afterwards a number of copies of the article, in which the necessary corrections had been made, but I was not able to provide every reader of the Journal with a copy. Only those who got the corrected offprint can make anything out of the analytic representation of the constructions. For the sake of those who have only the number of the Journal in hand, I may append here the corrected formulæ, page 398 ff.:

The conditions to be satisfied are that the saturation begins at a certain distance d from the centre to decrease in such a way that the length of the radius r and the arc of the corresponding angular value of the color-component, ϕ, are inversely proportional; or:

$$\text{When } r = d,\ \phi = 180^\circ;$$
$$\text{“}\quad r = a + d,\ \phi = 180 - \varepsilon;$$
$$\text{“}\quad r = na + d,\ \phi = 180 - n\varepsilon;$$

from which follows the equation of the curve

$$\phi = 180 - (r - d)\frac{\varepsilon}{a};$$

$$\text{and if we put } \frac{\varepsilon}{a} = \mu,$$

$$\phi = 180 - (r - d)\,\mu. \quad (1)$$

The value of μ is dependent on the size of the disc. If we wish to have the saturation 0 at the distance R from the centre, we must satisfy the special condition that

$$\phi = 0, \text{ when } r = R.$$

The above stated equation (1) takes, then, the form

$$\mu\,(R - d) = 180,$$

$$\text{or } \mu = \frac{180}{R - d}.$$

be desirable to produce all *tints* and *shades* of a color-tone, *i. e.*, all the transitions from white to the fully saturated color and from this to black, on one surface. In case we desire a continuous change in a simple arithmetical progression, this can be accomplished by a disc of the nature of that in Fig. 1, the simple construction of which scarcely requires any explanation. The inner part (I in our figure) is the very same as part A in Fig. 2 of the earlier article, and the outer part (III in our figure) is bounded by the continuation of the same common spirals, whose equation with reference to polar-coördinates could be derived as follows:

Let us call d the distance from the centre from which we wish to start, and R the radius of the disc, whilst the variables whose interdependence we are to express by the equation, *i. e.*, the radius at any place of the disc and the angular distance of the curve from the first radius may be denoted by r

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If we substitute this value for μ in the equation of the curve, we have

$$\phi = 180 - \frac{r-d}{R-d}\,180,$$

$$\text{or } \phi = 180\left(1 - \frac{r-d}{R-d}\right) \quad (2)$$

We have to determine now the equation for the curve which divides the remainder of the disc into a white and a black part. Suppose the intensity of the color was equal to that of a gray composed of n° white and m° black. The ratio of the white sector to the whole surface left by the color, then, will be $\frac{n}{n+m}$. And since the angular value of the whole uncolored surface must at any distance from the centre be $180^\circ - \phi$, the angular value of the white always will be

$$(180^\circ - \phi)\,\frac{n}{n+m};$$

or if we substitute the above stated value for ϕ,

$$\frac{180\,(r-d)}{R-d} \cdot \frac{n+m}{n}. \qquad (3)$$

In order to eliminate possible errors introduced by the spatial arrangement, it will be advisable to carry out each series of experiment with two discs, the one with the above stated arrangement, the other with the saturation increasing from the centre to the periphery. In this case the equations, corresponding to the above stated (2) and (3), read as follows:

$$\phi = \frac{180\,(r-d)}{R-r},$$

and the angular value of the white sector, the width of which is now decreasing from the centre to the periphery, can be expressed thus:

$$180\left(1 - \frac{r-d}{R-d}\right) \cdot \frac{n}{m+n}.$$

and φ respectively. The conditions to be satisfied by the equation, then, are:

$$\begin{aligned} \text{When } & r = d,\ \varphi = o; \\ \text{"} \quad & r = \chi + d,\ \varphi = y; \\ \text{"} \quad & r = n\chi + d,\ \varphi = ny; \quad (1) \\ \text{and in addition, "} \quad & r = R,\ \varphi = 360^\circ. \end{aligned}$$

From the third of these conditions it follows that

$$n = \frac{r - d}{\chi}.$$

If we substitute this value for n in the second part of the above statement (1), we have

$$\varphi = \frac{r - d}{\chi}\, y.$$

(Page 401 ff.)

If we wish to have an increase of the intensity in a geometrical progression from the centre to the circumference, the following conditions have to be satisfied:

$$\text{When } r = a,\ \phi = \varepsilon;$$
$$\text{when } r = na,\ \phi = \varepsilon^n;$$

$$\text{from which follows that } \phi = \varepsilon^{\frac{r}{a}}, \quad (1)$$

$$\text{or } r = a\,\frac{\log \phi}{\log \varepsilon}. \qquad (2)$$

In order to take into account the desired size of the disc, *i. e.*, in order to give ϕ a determined value X at a certain distance from the centre, we have to satisfy the condition that

$$r = R, \text{ when } \phi = X,$$

where R is the desired radius of the disc.

$$\text{We have, then, } X = \varepsilon^{\frac{R}{a}},$$

$$\text{or } \log X = \frac{R \log \varepsilon}{a},$$

$$\text{from which follows } \frac{a}{\log \varepsilon} = \frac{R}{\log X}.$$

If we substitute this value for $\frac{a}{\log \varepsilon}$ in the equations (1) and (2), we obtain

$$r = \frac{R \log \phi}{\log X},$$

$$\text{and } \phi = \sqrt[R]{X}. \qquad (3)$$

If, on the other hand, a decrease of intensity from the centre is desired, a deduction similar to that above stated leads from the

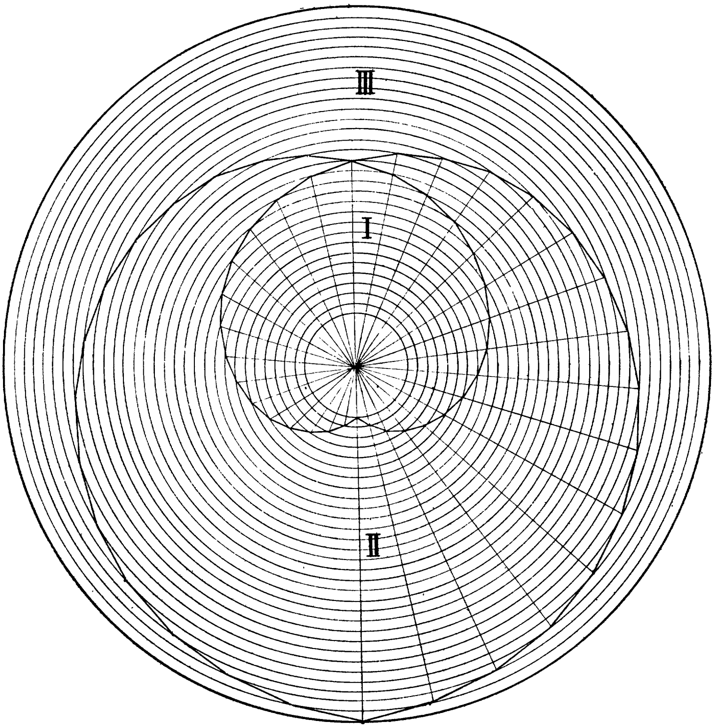

FIG. 1.

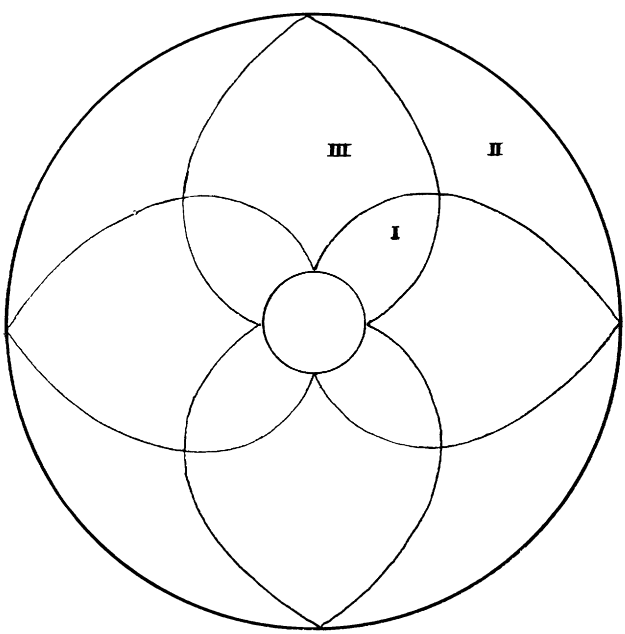

FIG. 2.

Let $\frac{y}{x}$ be called μ, then we have

$$\varphi = (r - d)\,\mu. \qquad (2)$$

In order to eliminate μ, which must be expressed in terms of d and R, we apply the formula just stated to the fourth of the conditions above. It reads, then,

$$360^\circ = (R - d)\,\mu;$$

$$\text{or} \quad \mu = \frac{360^\circ}{R - d}.$$

Substituting this value in our equation for the curve (2), we obtain

$$\varphi = \frac{(r - d)\,360^\circ}{R - d}. \qquad (3)$$

In practically applying the above results we have to cover part I of the disc with white, part II with black (black velvet), whilst part III has to be occupied by the color whose tints and shades we wish to produce. A disc of this construction will, when in rotation, show from the centre to the middle of the radius all tints from white to the full saturation of the pigment applied, and from these to the periphery all shades from full saturation to black in continuous transition. Similarly, a disc with black at part I and white at part II will show the very same in opposite order.

In order to facilitate the blending of the components, it may be recommended that the construction should be applied, not for the whole disc, but, perhaps, for each third or quarter of it. Fig. 2 gi‹es the appearance of a disc, which has the above construction for each quadrant.

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condition: when $r = na$, $\phi = \sqrt[n]{\varepsilon}$ to the equation

$$\phi = \sqrt[r]{X^R}, \qquad (4)$$

where R denotes, as above, the radius of the disc and X the desired angular value of the white at the circumference.

(Page 403 ff.)

$$\text{When } r = na,\ k\,\phi + (180 - \phi) = \varepsilon^n,$$
$$\text{or } \phi\,(k - 1) + 180 = \varepsilon^n,$$

which, treated as in the simpler case above, leads to the equation

$$\phi\,(k - 1) + 180 = \sqrt[R]{[X\,(k - 1) + 180]^r},$$

$$\text{or } \phi = \frac{\sqrt[R]{[X\,(k-1) + 180]^r} - 180}{k - 1}.$$

It will easily be seen that, if a logarithmic increase and decrease is desired, the formulæ according to which the disc in Fig. 7 of the former publication is constructed, *i. e.*,

$$\varphi = \frac{r}{\sqrt{X}} \quad \text{and} \quad \varphi = \frac{R}{\sqrt{X^{r}}}$$

require only little modification to adapt them to the present purpose.

It may be mentioned, also, that the above described construction may be utilized for demonstrating the totality of possible mixtures between a certain color-tone and a pair of complementary colors; *e. g.*, if we wish to have the transition from yellow to its complementary, violet, not through grey, but through red, we have to cover part III of the above disc with red, and parts I and II with yellow and violet (or *vice versa*) respectively.